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Control and Control Theory for Flexible Robots

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Abstract

As the requirements for robot performance increase, the dynamics of the manipulator become more dominated by flexibility. These flexible effects generate model uncertainty which reduces the end-point positioning accuracy of the manipulator. Residual vibration or tip deflection due to uncertain payloads may contribute to the error in tip position. This paper addresses several control strategies currently used by researchers to account for flexibility in robots and their ability to perform tasks despite the flexibility.

1. Introduction

The demands for increased robot accuracy coupled with high speed and large workspace requirements necessitate the evaluation of robot flexibility. The influence of flexibility on modelling and controller design must be better understood to achieve these requirements. This paper presents a review of current research in the control of flexible robots and reports on algorithms with specific experimental and theoretical results. Other researchers have conducted such surveys regarding modelling, design and control of flexible robot arms. Desoyer, Kopacek, Lugner and Troch [24] compare various modelling methods for lightweight robots and discuss the effects of flexibility on possible control strategies. They examine the kinetostatic method, the vibrational mode approach and the finite element method as a means of modelling flexible systems. Troch and Kopacek [60] discuss control strategies for flexible robots, designs based on model simplification and the effects of actuator dynamics. Peng and Liou [43] survey experimental studies involving flexible mechanisms from a designer's point of view. They examine the identification of damping and mode shapes, vibration reduction and various means of measuring flexible mechanism responses. Book [11] describes the modelling of flexibility, the large motion equations used and the design of flexible arms. He also presents trajectory planning and trajectory tracking strategies for the control of flexible robot arms.

This paper begins with a short discussion of modelling the behavior of a flexible beam. The mathematical expression for the beam deflection is given as a function of mode shapes and generalized coordinates. The difficulty in choosing appropriate mode shapes and representing the correct boundary conditions is then presented. Once an expression for the beam deflection is established, it can be incorporated into a recursive Lagrangian approach for modelling the dynamics of a serial chain of flexible links. The dependence of several control algorithms on model information is then investigated. The control methods surveyed involve end-point tracking, trajectory planning, modal damping and vibration suppression. Several model identification algorithms are also presented to demonstrate adaptive control for flexible systems with variable payload.

2. Modelling Flexible Arms

The analysis of a flexible system begins with developing a model that describes the position of each point in the system relative to a suitable inertial reference frame. The model can then be used to simulate the system response to various inputs or to synthesize a larger, more complex system. The mathematical description of a flexible manipulator is often approximated as a set of serially connected links with proper boundary conditions. During robot motion, each flexible link can experience torsion, deflection or elongation. Torsion about the longitudinal axis of the link has little effect in the overall dynamics if the height of the link is large compared to its width. However, these dimensions permit a significant deflection parallel to the width direction of the link that must be considered. The elongation or compression of a flexible link is often neglected since the axial stiffness is sufficient to prevent such a motion during robot manipulation. Therefore, modelling the transverse deflection of a flexible link is sufficient to describe the dynamics of a slender, flexible member.

The dynamics of the link deflection can be represented by the Euler-Bernoulli beam equation

$$\rho A \frac{\partial^2 y(x,t)}{\partial t^2} + EI \frac{\partial^4 y(x,t)}{\partial x^4} = f(x,t) \quad (1)$$

which neglects shearing of the beam and rotational inertia of a differential element about the longitudinal axis. The terms in the beam equation are the mass per unit length - ρA, the stiffness of the link - EI and the external load per unit length - $f(x,t)$. This partial differential equation is adequate to represent the dynamic solution for the slender links used in the simplified serial robot. The solution of such an equation involves the separation of time and space with one dependent spatial coordinate representing the deflection of the link.

If time is treated as a continuous variable, an ordinary differential equation results when the beam deflection is represented as an infinite sum of basis functions, each multiplied by a time-varying amplitude. For all practical purposes, the beam deflection can be approximated with a finite sum similar to Equation (2)

$$y(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t) \quad (2)$$

where the ϕ_i are the assumed mode shapes and the q_i are the generalized modal coordinates describing the deflection. Hughes [30] offers a relevant discussion on mode determination.

A wide variety of choices exist to determine possible mode shapes. They can be simple polynomials, eigenfunctions that result from eigenvalue problems, eigenvectors from finite element analysis or mode shapes determined from experimental data. The only constraint is that the mode shapes must satisfy the set of boundary conditions imposed by the physical system. Bellezza, Lanari and Ulivi [6] formulated pseudo-clamped and pseudo-pinned boundary value problems to show the models differed only by a nodal transformation. They also verified their dynamic models experimentally and found errors in eigenfrequency of less than 8%. Forrest-Barlach [25] considered a flexible robot arm as a nonhomogeneous, time-varying boundary condition problem which is then transformed into a homogeneous problem with distributed input forces. Oakley and Cannon [40] experimentally determined system mode shapes for the assumed modes method using photographic techniques. Using this information, they compared theoretical mode shapes to develop a low-order model that accurately represents the two-link, flexible system.

Once the link deflection can be accurately described, 4 X 4 transformation matrices can be used to find the dynamics of flexible robots. Book [9] enhances the 4 X 4 rigid arm formulation of Denavit and Hartenberg [23] to include the deflection of each link. The method is a Lagrangian-assumed modes approach and produces a dynamic model that is similar in form to the rigid manipulator model. Therefore, the flexible effects are easily distinguished in the dynamic model for easy transformation to flexible arm control using rigid arm techniques. This Lagrangian approach was implemented symbolically for compliant joints and links but was not recursive in nature [15].

3. Controlling Flexible Arms

The primary objectives of flexible arm control are accurate end-point positioning while a given task is being performed and robustness to any unmodelled dynamics. The amount of flexibility in the manipulator may be beneficial for some tasks, e.g. force generation in a bracing robot, and may be detrimental in others, e.g. rapid trajectory control, unknown payload positioning. This discussion will explore many different control strategies for positioning flexible robots. This survey is by no means complete but is only meant to give a flavor for the various flexible arm control algorithms.

3.1 End-Point Control

In most flexible robot applications, the sensors are not collocated with the actuators which results in a nonminimum phase system due to the flexibility. Spector and Flashner [58] investigated the sensitivity of right-half plane zeros inherent to nonminimum phase systems and found that the frequency at which a transfer function becomes nonminimum phase decreases as the sensor/actuator distance increases. Park and Asada [42] have conducted work to minimize the sensor/actuator distance to yield a minimum phase end-point control system. They attached a special transmission mechanism to change the location of the torque application point along the beam. Experiments verify the nonminimum phase behavior of the link.

Davis and Hirschorn [20] formulated a hybrid lumped/distributed model for the tracking control of a single, flexible link. Their design consists of coaxial mounted beams to improve the response of the link. By collocating a force actuator between the tips of the beams, near perfect tracking is achieved using acceleration feedback. The tracking performance is theoretically justified but no experimental results verify their claims.

Cannon and Schmitz [14] conducted experiments to demonstrate precise end-point control of a single link, flexible manipulator. They showed that a limit in control bandwidth exists due to a wave propagation delay along the beam. Their LQG (Linear Quadratic Guassian) controller design was verified with a step command in desired tip position. The nonminimum phase characteristics are evident in the time response of the tip, specifically the negative displacement of the tip for a positive input at the hub. The bandwidth of the controller was later increased with the addition of a wrist attached to the tip of the flexible manipulator [18]. Rapid pick-and-place tasks were performed without stopping the motion of the flexible arm. Ballhaus and Rock [3] extended this work to incorporate two flexible links with a mini-manipulator mounted at the tip.

The previous end-point tracking techniques attempt to transform the nonminimum phase nature of noncollocated flexible systems to achieve a desired tracking performance. The next few methods involve computing the necessary joint torques to produce a specified tip motion. A very accurate mathematical relationship between tip position and joint torque is required for these methods due to their strictly feedforward nature. Bayo [4] proposed an inverse dynamics problem that calculates the input forces necessary to move the tip of a flexible robot arm along a prescribed path. The original computed torque method was computationally burdensome requiring Fourier and inverse Fourier transforms to calculate the time domain torque input. Bayo and Moulin [5] later simplified the method by introducing a convolution integral method to solve the inverse dynamic equations in real time.

Asada, Ma and Tokumaru [2] pursued the inverse dynamics problem using virtual rigid link coordinates. This coordinate system transformation simplifies the computation of the inverse dynamic equations and also yields simplified boundary conditions. Simulation results compare the rigid model to the computationally efficient flexible model which produces a much smaller tracking error.

Kwon and Book [32] later supplied a simple inverse dynamics solution that computes the required torque in the time domain. Their inverse dynamic method also generates the state trajectories that make the tip of the flexible robot follow the desired tip trajectory. Experimental results verify simulation to produce a control method that provides good tracking performance without tip overshoot. Recently, they extended the inverse dynamics method to contact control using flexible systems [12]. Their research addresses important issues involving tracking control during free motion, the transition to contact with the environment and contact force regulation.

A relatively new area of end-point control research deals positioning the tip of a flexible manipulator with variable payload. Harashima, Nishiyama, Ueshiba and Hashimoto [27] detected the tip of a manipulator with a CCD (Charge Coupled Device) camera mounted on the hub of the arm. Using an adaptive AR (AutoRegressive) model, they achieved fast positioning of the tip without overshoot. Obergfell and Book [41] fixed the position of a CCD camera relative to the workspace of a flexible robot and sensed its position using retroreflective landmarks. They achieved an average positioning error of just 0.03" in a 20' long arm using a 55 lb. payload.

Payload estimation permits accurate positioning of the flexible arm tip with the use of cameras. Chirinos, Shusterman, Gonzalez and Widmann [19] used an adaptive scheduling control to position a flexible manipulator in the presence of payload variation. A look-up table of possible controller parameters versus possible tip payloads proved promising in simulation as a future gain scheduling algorithm. Nelson and Mitra [38] provide an on-line estimator of payload to adjust controller gains that maintains a desired arm response. Simulation results demonstrate accurate load estimations during accurate arm movements.

3.2 Joint Control

Most flexible robots are positioned using independent joint controllers with the end-point estimated by a kinematic relation. Goldenberg [26] developed a feedforward controller along with a PD (Proportional-Derivative) feedback controller to control the tip position of a flexible arm. The method calculates a nominal torque required for a given trajectory based on a rigid body model of the robot. A feedforward term provides pole-zero cancellation of the dominant closed loop poles of the system while the feedback eliminates any error in the joint variable. Later, he demonstrated that the feedback control also supplies robustness to plant parameter variations [52]. Simulation results verify the claims of a very effective control method for a single link, flexible arm.

The control of combined rigid and flexible motion systems actually began much earlier. Book, Maizza-Neto and Whitney [8] considered modal control as a means of accommodating link flexibility by comparing various joint and flexible state feedback configurations. IJC (Independent Joint Control) was found to provide a control bandwidth of up to one-half the first natural frequency of the system with clamped joint angles. GRC (General Rigid Control) showed twice the bandwidth of IJC with little increase in complexity. However, FFC (Flexible Feedback Control) required too much computation to be useful in actual implementation at that time. The computational speed of current hardware now makes FCC possible.

The method of mode suppression appears in many different controller forms but the objective remains the same, accurate joint positioning while attempts are made to minimize elastic motion. Singh and Schy [57] decoupled the rigid and flexible motion which allowed for joint angle control independent of elastic motion stabilization. The joint angles are controlled by torques generated from the nonlinear inversion of the rigid system while the flexible motion is controlled with force controllers. Nathan and Singh [37] later controlled the flexible motion without the use of external force stabilization. Their control method defines a sliding surface that provides accurate joint control to within a specified neighborhood of the final state. Using switching logic, a modal stabilization algorithm damps the vibration to allow final positioning of the tip. Simulation results demonstrate the effectiveness of each phase of the algorithm to achieve the final positioning accuracy.

Another approach to separate flexible and rigid motion is based on their fast and slow motion. Singular Perturbation theory provides analysis of two time scale systems. Spong, Khorasani and Kokotovic [59] applied the technique to compliant joint robots, Siciliano and Book [54] controlled a flexible manipulator and Book and Lee [10] generated inertial forces with a small, rigid robot to control a flexible robot. Care must be taken in design to ensure a distinct separation of fast and slow dynamics for the theory to hold. De Maria and Siciliano [22] extend the approach to a number of fast controllers, one for each assumed mode of the flexible model.

Other researchers have used similar control partitioning schemes to control flexible robots. Chalhoub and Ulsoy [17] devised a rigid and flexible motion controller that significantly reduced the dynamic deflection of a flexible arm. Using strain feedback, additional damping of the flexible motion is provided to improve the positioning accuracy of the end-effector. Experimental results show a reduction in maximum deflection with a much smaller settling time.

Schoenwald, Feddema, Eisler and Segalman [51] describe a LQG regulator that determines feedback gains for position, velocity and strain for straight line positioning. The feedback is added to a computed torque feedforward term to yield better results than either strategy alone. Experiments showed promising results but the authors stated that improvement on the linearized finite-element model and LQG design are needed. Biswas and Klafter [7] devise an optimum regulation control scheme that achieves a desired angular rotation while suppressing the vibration of a flexible manipulator. Using Pontryagin's minimum principle on a desired performance index, the optimal control effort is found. Using three mode shapes, simulation results show improved positioning accuracy of the end-effector on a flexible arm.

De Luca and Siciliano [21] compare open vs. closed-loop strategies for joint feedback control. They found that open-loop joint torque plus PD joint feedback provides satisfactory performance with respect to final joint error. Oscillations in final tip position were also reduced in simulation when the natural passive damping of the flexible arm is considered.

A combination of feedforward and feedback loops can also be used for tip position control. Rattan, Feliu and Brown [45] created a partitioning scheme that divided the control effort into a model-based portion and a servo portion to increase tip positioning accuracy of a flexible arm. A feedforward term is also added to prevent delay in the arm's response to the desired trajectory. Reisenauer, Balas and Ramey [47] implemented a ROM (Reduced-Order Model) controller to position a large, flexible manipulator. To reduce unmodelled dynamic effects, a RMF (Residual Mode Filter) is used to prevent spillover of the higher-order dynamics and to create a stable feedback control system. Experiments verify that the ROM controller alone is unstable and the addition of the RMF is required to stabilize the system.

ROMs are also used to adapt the control effort when the physical system motion differs from the model output. The model reference adaptive control (MRAC) technique excites both the flexible manipulator and a mathematical reference model with the same input. The difference in the two outputs generates an error term that is used for adaptive control. Siciliano, Yuan and Book [53] implemented the controller on a single flexible link and improvements were obtained over a pure optimal control regulator. Sasiadek and Srinivasan [50] applied the MRAC method to a simulated, two-link, flexible manipulator with comparable positioning results.

Many researchers use a much simplified model to control a flexible manipulator. The model consists of a series of connected spring-mass-damper subsystems with the control objective being to position the last degree of freedom. Bridges, Zhu, Dawson and Qu [13] implement this modelling technique for a robust controller design that achieves global uniform ultimate boundedness for the tracking error in the presence of unknown model parameters. By limiting the reference model transfer function to be minimum phase, the tracking error can be made arbitrarily small by adjusting the controller gains as verified in their simulation results. Sardar and Paul [49] use a similar model with modal feedback to drive the poles corresponding to the flexible dynamics far into the left-half plane. This effectively damps out the vibration and is verified with simulation output. However, the method has not been verified experimentally.

Alberts, Book and Dickerson [1] experimented with a constraining layer of viscoelastic material to help damp vibration associated with a modal feedback control algorithm. Preumont, Dufour and Malekian [44] collocated force transducers with piezoelectric actuators to eliminate vibration in large space structures. The control strategy performs a -90° phase shift between the measured force and the actuators. Matsuno and Sakawa [35] developed an equivalent spring model to represent all the flexibility in a six degree of freedom, flexible manipulator. Using acceleration feedback, a state feedback controller estimated future states and vibration was suppressed. Experimental trajectories verify the reduction in vibration amplitude.

3.3 Vibration Suppression

After reviewing end-point and joint control algorithms, the difficulty in accurate end-point positioning of a flexible manipulator is eliminating the residual vibration. Singer and Seering [55] developed a input shaping method that eliminates end-point residual vibration in a feedforward manner. A desired trajectory is convolved with an impulse sequence to produce a new trajectory that does not excite the resonances of the flexible system. The method provides excellent vibration suppression for constant parameter systems and was verified on single degree of freedom systems. Noakes and Jansen [39] implemented a similar method to position suspended payloads in a nuclear waste handling operation. The oscillating motion of suspended objects was eliminated and accurate positioning of the payloads was possible.

Other researchers have tried to extend Singer and Seering's work to more complex systems. Zuo and Wang [61] proposed a closed-loop input shaping method to control a large class of multi-link manipulators with one flexible link. Stability of the controller design is discussed and experiments verify the implementation. Rattan and Feliu [46] derived a feedforward controller based on the dynamics model inversion technique. They show that Singer and Seering's method is a special case of their method when the delay is chosen to be one-half the damped natural period. However, their method produces shorter rise times for the tip response of a flexible arm.

Hillsley and Yurkovich [29] used input shaping for large angle movements in a two-link arm but found that residual vibration remained after commanded motion ceased. They included end-point acceleration feedback to eliminate the residual vibration. The problem occurred because the input shaping algorithm can not accommodate large variations in natural frequency associated with the modes of vibration. Magee and Book [33] created a MCF (Modified Command Filtering) technique to accommodate time-varying parameters of a flexible system to eliminate residual vibration. Their derivation works for a large variation in natural frequency because the filter adjusts to the time-varying nature of the flexible system. Using the MCF method in a joint feedback control loop, initial experiments with a large two-link, flexible manipulator demonstrate excellent vibration suppression capabilities. Meckl and Seering [36] have also attempted to reduce residual vibration in the presence of time-varying resonances. They established a relationship between the input function and the resulting residual vibration acceleration to minimize the spectral magnitude about a system resonance. Simulation results showed the amplitude of acceleration response was nearly eliminated for small variations in system frequency. Singer and Seering [56] created a frequency sampling method in an attempt to suppress the residual vibration of time-varying systems. However, nonlinear inequality constraints must be satisfied along with solving nonlinear, trigonometric equations which makes the method less attractive.

The previous discussion was limited to vibration suppression of just the first mode of vibration in flexible systems. Hyde and Seering [31] have extended Singer's work to multiple mode vibration suppression. However, their solution involves solving a large set of nonlinear, trigonometric equations. Magee and Book [34] recently expanded their MCF method to eliminate two modes of vibration on the same large manipulator. Their filtering method is iterative in nature and does not require the solution of nonlinear equations. Experimental results verify multiple mode reduction with improved results over Singer's method. The improvement in vibration suppression ability is again due to the time-varying nature of the filter. Future work of the authors is to investigate on-line identification of the modal properties of the flexible system similar to other work [16,28,48] and to make the MCF technique robust with respect to payload uncertainty.

4. Conclusions

The consideration of flexibility in robot arm control can no longer be ignored. The demands for large, light-weight manipulators for space and domestic applications require the need for better flexible control algorithms. Many new approaches in modelling and control of flexible manipulators are being investigated and this survey attempted to address several of the algorithms. Emphasis was placed on experimental support of theoretical and simulation results to remind us all of the real world applications associated with the outcome of research.

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